Introduction

This report focuses on DC to AC power inverters, which aim to efficiently transform a DC power source to a high voltage AC source, similar to power that would be available at an electrical wall outlet. Inverters are used for many applications, as in situations where low voltage DC sources such as batteries, solar panels or fuel cells must be converted so that devices can run off of AC power. One example of such a situation would be converting electrical power from a car battery to run a laptop, TV or cell phone.

The method, in which the low voltage DC power is inverted, is completed in two steps. The first being the conversion of the low voltage DC power to a high voltage DC source, and the second step being the conversion of the high DC source to an AC waveform using pulse width modulation. Another method to complete the desired outcome would be to first convert the low voltage DC power to AC, and then use a transformer to boost the voltage to 120 volts. This project focused on the first method described and specifically the transformation of a high voltage DC source into an AC output.

Of the different DCAC inverters on the market today there are essentially two different forms of AC output generated: modified sine wave, and pure sine wave. A modified sine wave can be seen as more of a square wave than a sine wave; it passes the high DC voltage for specified amounts of time so that the average power and rms voltage are the same as if it were a sine wave. These types of inverters are much cheaper than pure sine wave inverters and therefore are attractive alternatives.

Pure sine wave inverters, on the other hand, produce a sine wave output identical to the power coming out of an electrical outlet. These devices are able to run more sensitive devices that a modified sine wave may cause damage to such as: laser printers, laptop computers, power tools, digital clocks and medical equipment. This form of AC power also reduces audible noise in devices such as fluorescent lights and runs inductive loads, like motors, faster and quieter due to the low harmonic distortion.
Objective

In the market of power inverters, there are many choices. They range from the very expensive to the very inexpensive, with varying degrees of quality, efficiency, and power output capability along the way. High quality combined with high efficiency exists, though it is often at a high monetary cost.

The high end pure sine wave inverters tend to incorporate very expensive, high power capable digital components. The modified sine wave units can be very efficient, as there is not much processing being performed on the output waveform, but this results in a waveform with a high number of harmonics, which can affect sensitive equipment such as medical monitors. Many of the very cheap devices output a square wave, perhaps a slightly modified square wave, with the proper RMS voltage, and close to the right frequency.

Our goal is to fill a niche which seems to be lacking in the power inverters market, one for a fairly efficient, inexpensive inverter with a pure sine wave output. Utilizing PWM and analog components, the output will be a clean sinusoid, with very little switching noise, combined with the inexpensive manufacturing that comes with an analog approach.
Background

**DC and AC Current:** In the world today there are currently two forms of electrical transmission, Direct Current (DC) and Alternating Current (AC), each with its own advantages and disadvantages. DC power is simply the application of a steady constant voltage across a circuit resulting in a constant current. A battery is the most common source of DC transmission as current flows from one end of a circuit to the other. Most digital circuitry today is run off of DC power as it carries the ability to provide either a constant high or constant low voltage, enabling digital logic to process code executions. Historically, electricity was first commercially transmitted by Thomas Edison, and was a DC power line. However, this electricity was low voltage, due to the inability to step up DC voltage at the time, and thus it was not capable of transmitting power over long distances.

\[ V = IR \]
\[ P = VI = I^2R \] \( (1) \)

As can be seen in the equations above, power loss can be derived from the electrical current squared and the resistance of a transmission line. When the voltage is increased, the current decreases and concurrently the power loss decreases exponentially; therefore high voltage transmission reduces power loss. For this reasoning electricity was generated at power stations and delivered to homes and businesses through AC power. Alternating current, unlike DC, oscillates between two voltage values at a specified frequency, and its ever changing current and voltage makes it easy to step up or down the voltage. For high voltage and long distance transmission situations all that is needed to step up or down the voltage is a transformer. Developed in 1886 by William Stanley Jr., the transformer made long distance electrical transmission using AC power possible.

Electrical transmission has therefore been mainly based upon AC power, supplying with a 230 volt AC source. It should be noted that since 1954 there have been many high voltage DC transmission systems implemented around the globe with the advent of DC/DC converters, allowing the easy stepping up and down of DC voltages.
Inverters & Its Applications

Power inverters are devices which can convert electrical energy of DC form into that of AC. They come in all shapes and sizes, from low power functions such as powering a car radio to that of backing up a building in case of power outage. Inverters can come in many different varieties, differing in price, power, efficiency and purpose. The purpose of a DC/AC power inverter is typically to take DC power supplied by a battery, such as a 12 volt car battery, and transform it into a 230 volt AC power source operating at 50 Hz, emulating the power available at an ordinary household electrical outlet.

Figure: Commercial Inverter

Figure provides a idea of what a small power inverter looks like. Power inverters are used today for many tasks like powering appliances in a car such as cell phones, radios and televisions. They also come in handy for consumers who own camping vehicles, boats and at construction sites where an electric grid may not be as accessible to hook into. Inverters allow the user to provide AC power in areas where only batteries can be made available, allowing portability and freeing the user of long power cords.
On the market today are two different types of power inverters, modified sine wave and pure sine wave generators. These inverters differ in their outputs, providing varying levels of efficiency and distortion that can affect electronic devices in different ways.

A modified sine wave is similar to a square wave but instead has a “stepping” look to it that relates more in shape to a sine wave. This can be seen in Figure 2, which displays how a modified sine wave tries to emulate the sine wave itself. The waveform is easy to produce because it is just the product of switching between 3 values at set frequencies, thereby leaving out the more complicated circuitry needed for a pure sine wave. The modified sine wave inverter provides a cheap and easy solution to powering devices that need AC power. It does have some drawbacks as not all devices work properly on a modified sine wave, products such as computers and medical equipment are not resistant to the distortion of the signal and must be run off of a pure sine wave power source.

![Figure: Square, Modified, and Pure Sine Wave](image)

Pure sine wave inverters are able to simulate precisely the AC power that is delivered by a wall outlet. Usually sine wave inverters are more expensive than modified sine wave generators due to the added circuitry. This cost, however, is made up for in its ability to provide power to all AC electronic devices, allow inductive loads to run faster and quieter, and reduce the audible and electric noise in audio equipment, TV’s and fluorescent lights.


➤ **Types of Inverter**

Depending upon the input and output there are two types of inverter .... (a) Voltage Source Inverter (VSI) & (b) Current Source Inverter (CSI).

(a) **Voltage Source Inverter (VSI):** In these type of inverter input voltage is maintained constant and the amplitude of output voltage does not depend on the load. However, the waveform of load current as well as its magnitude depends upon the nature of the load impedance.

(b) **Current Source Inverter (CSI):** In these type of inverter input current is constant but adjustable. The amplitude of output current from CSI is independent of load. However the magnitude of output voltage and its waveform output from CSI is dependent upon the nature of load impedance. A CSI does not require any feedback diodes whereas these are required in VSI.

![Voltage Source Inverter (VSI)](image1.png) ![Current Source Inverter (CSI)](image2.png)
- **Internal Control of Inverter**

Output voltage from an inverter can also be adjusted by exercising a control within the inverter itself. The most efficient method of doing this is by pulse-width modulation control used within an inverter. This is discussed briefly in what follows:

**Pulse Width Modulation Control:**

In electronic power converters and motors, PWM is used extensively as a means of powering alternating current (AC) devices with an available direct current (DC) source or for advanced DC/AC conversion. Variation of duty cycle in the PWM signal to provide a DC voltage across the load in a specific pattern will appear to the load as an AC signal, or can control the speed of motors that would otherwise run only at full speed or off. This is further explained in this section. The pattern at which the duty cycle of a PWM signal varies can be created through simple analog components, a digital microcontroller, or specific PWM integrated circuits.

![PWM Technique and Block diagram](image)

Figure: PWM Technique and Block diagram
The top picture shows the input reference waveform (square wave) and a carrier wave (triangular wave) is passed into a comparator to achieve the PWM waveform. The triangular wave is simple to create, utilizing an opamp driver. The triggering pulses are generated at the points of intersection of the carrier and reference signal waves. The firing pulses are generated to turn-on the SCRs so that the output voltage is available during the interval triangular voltage wave exceeds the square modulating wave.

The advantages possessed by PWM technique are as under:

(i) The output voltage control with this method can be obtained without any additional components.

(ii) With this method, lower order harmonics can be eliminated or minimised along with its output voltage control. As higher order harmonics can be filtered easily, the filtering requirements are minimized.

The main disadvantage of this method is that the SCRs are expensive as they must possess low turn-off and turn-on times.

Different PWM techniques are as under:

(a) Single-pulse modulation
(b) Multiple-pulse modulation
(c) Selected harmonic elimination (SHE) PWM
(d) Minimum ripple current PWM
(e) Sinusoidal-pulse PWM (SPWM)
(f) Space vector-pulse PWM (SVPWM)

Mainly SPWM & SVPWM is used in industry and domestic uses.

(a) **Single Pulse Modulation:** When the waveform of output voltage from single-phase full-bridge inverter is modulated. It consists of a pulse of width 2d located symmetrically about π/2 and another pulse located symmetrically about 3π/2. The range of pulse width 2d varies from 0 to π; i.e. 0<2d<π. The output voltage is controlled by varying the pulse width 2d. This shape of the output voltage wave is called quasi-square wave.
(b) **Multiple Pulse Modulation:** This method of pulse modulation is an extension of single-pulse modulation. In this method, several equidistant pulses per half cycle are used. For simplicity, the effect of using two symmetrically spaced pulses per half cycle is investigated here.

(c) **Selected Harmonic Elimination (SHE) PWM:** The undesirable lower order harmonics of a square wave can be eliminated and the fundamental voltage can be controlled as well by what is known as selected harmonic elimination (SHE) PWM. In this method, notches are created on the square wave at predetermined angles. In the figure, positive half cycles output is shown with quarter-wave symmetry. It can be shown that the four notch angles $\alpha_1, \alpha_2, \alpha_3$ & $\alpha_4$ can be controlled to eliminate three significant harmonic components and control the fundamental voltage. A large no. of harmonics can be eliminated if the waveform can accommodate additional notch angles.

The general Fourier series of the wave can be given as

$$v(t) = \sum_{n=1}^{\infty} \left( a_n \cos n\omega t + b_n \sin n\omega t \right)$$

![Phase Voltage Wave for SHEPWM](image)
Where

\[ a_n = \frac{1}{\pi} \int_0^{2\pi} v(t) \cos n\omega_0 t \, dt \]

\[ b_n = \frac{1}{\pi} \int_0^{2\pi} v(t) \sin n\omega_0 t \, dt \]

For a waveform with quarter-cycle symmetry only the odd harmonics with sine components will be present. Therefore,

\[ a_n = 0 \]

\[ v(t) = \sum_{n=1}^{\infty} b_n \sin n\omega_0 t \]

Where,

\[ b_n = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} v(t) \sin n\omega_0 t \, dt \]

Assuming that the wave has unit amplitude that is \( v(t) = 1 \), \( b_n \) can be expanded as

\[ b_n = \frac{4}{\pi} \left[ \int_0^{\alpha_1} (+1) \sin n\omega_0 t \, dt + \int_{\alpha_1}^{\alpha_2} (-1) \sin n\omega_0 t \, dt + \int_{\alpha_2}^{\alpha_3} (+1) \sin n\omega_0 t \, dt + \cdots + \int_{\alpha_{n-1}}^{\alpha_n} (-1)^{K-1} \sin n\omega_0 t \, dt + \int_{\alpha_n}^{\frac{\pi}{2}} (+1) \sin n\omega_0 t \, dt \right] \]

Using the general relation

\[ \int_{\theta_1}^{\theta_2} \sin n\omega_0 t \, dt = \frac{1}{n} \left( \cos n\theta_1 - \cos n\theta_2 \right) \]

The last and first terms are

\[ \int_0^{\alpha_1} (+1) \sin n\omega_0 t \, dt = \frac{1}{n} \left( 1 - \cos n\alpha_1 \right) \]

\[ \int_{\frac{\pi}{2}}^{\alpha_n} (-1) \sin n\omega_0 t \, dt = \frac{1}{n} \cos n\alpha_n \]
Thus we get,

\[ b_n = \frac{4}{n\pi} \left[ 1 + 2 \left( -\cos n\alpha_1 + \cos n\alpha_2 - \ldots + \cos n\alpha_K \right) \right] \]

\[ = \frac{4}{n\pi} \left[ 1 + 2 \sum_{k=1}^{K} (-1)^{k} \cos n\alpha_k \right] \]

Consider, for example the 5\textsuperscript{th} & 7\textsuperscript{th} harmonics (lowest significant harmonics) are to be eliminated and the fundamental voltage is to be controlled. The 3\textsuperscript{rd} and other triplen harmonics can be ignored if the machine has an isolated neutral. In this case, K=3 and the simultaneous equation can be written.

Fundamental:

\[ b_1 = \frac{4}{\pi} \left( 1 - 2 \cos \alpha_1 + 2 \cos \alpha_2 - 2 \cos \alpha_3 \right) \]

5\textsuperscript{th} harmonic:

\[ b_5 = \frac{4}{5\pi} \left( 1 - 2 \cos 5\alpha_1 + 2 \cos 5\alpha_2 - 2 \cos 5\alpha_3 \right) = 0 \]

7\textsuperscript{th} harmonic:

\[ b_7 = \frac{4}{7\pi} \left( 1 - 2 \cos 7\alpha_1 + 2 \cos 7\alpha_2 - 2 \cos 7\alpha_3 \right) = 0 \]
As the fundamental frequency decreases the no. of notch angles can be increased so that a higher no. of significant harmonics can be eliminated. Again the number of notch angles/cycles or the switching frequency can be determined by the switching loss of inverter. An obvious disadvantage of the scheme is that the lookup table at low fundamental frequency is unusually large. For this reason, a hybrid PWM scheme where the low frequency, low voltage region uses SPWM method.

**(d) Minimum Ripple Current PWM:** One disadvantage of the SHE PWM method is that the elimination of lower order harmonics considerably boosts the next higher level of harmonics. Since the harmonic loss in a machine is dictated by the RMS ripple current, it is this parameter that should be minimized instead of emphasizing the individual harmonics. The effective leakage inductance of a machine essentially determines the harmonic current corresponding to any harmonic voltage. Therefore, the expression of RMS ripple current can be given as:
\[
I_{\text{ripple}} = \sqrt{I_5^2 + I_7^2 + I_11^2 + \ldots} \\
= \sqrt{\frac{\hat{I}_5^2}{2} + \frac{\hat{I}_7^2}{2} + \frac{\hat{I}_11^2}{2} + \ldots} \\
= \sqrt{\frac{1}{2} \sum_{n=5,7,11,\ldots}^{\infty} \left( \frac{V_n}{n\omega L} \right)^2}
\]

Where

\[I_5, I_7, \ldots = \text{RMS Harmonic Current}\]
\[L = \text{Effective Leakage Inductance of the Machine}\]
\[\hat{I}_5, \hat{I}_7, \ldots = \text{Peak value of Harmonic Current}\]
\[n = \text{Order of Harmonics}\]
\[V_n = \text{Peak Value of nth Order Harmonic}\]
\[\omega = \text{Fundamental Frequency}\]

The corresponding harmonic copper loss

\[P_l = 3I_{\text{ripple}}^2 R\]

Where \(R = \text{Effective per phase resistance of the machine}\)

\[\text{(e) Sinusoidal Pulse Width Modulation:}\] The most common and popular technique of digital pure-sine wave generation is sinusoidal pulse-width-modulation (SPWM). The SPWM technique involves generation of a digital waveform, for which the duty-cycle is modulated such that the average voltage of the waveform corresponds to a pure sine wave. The simplest way of producing the SPWM signal is through comparison of a low-power reference sine wave \(v_r\) with a high frequency triangle wave \(v_c\). Using these two signals as input to a comparator, the output will be a 2-level SPWM signal. The intersection of \(v_r\) & \(v_c\) waves determines the switching instants and commutation of the modulated pulse. In figure \(V_c\) is the peak value of triangular carrier wave and \(V_r\) that of the reference or modulated signal.
The carrier and reference waves are mixed in a comparator. When sinusoidal wave has magnitude higher than the triangular wave, the comparator output is high, otherwise it is low. The comparator output is processed in a trigger pulse generator in such a manner that the output voltage wave of the inverter has a pulse width in agreement with the comparator output pulse width.

When triangular carrier wave has its peak coincident with zero of the reference sinusoid, there are $N=f_c/2f$ pulses per half cycle; Fig.8 has five pulse. In case zero of the triangular wave of coincides with zero of the reference sinusoid, there are $(N-1)$ pulses per half cycle.
Fig. - principle of SPWM

Fig. - Line and phase voltage waves of PWM inverter
Modulation index:

The ratio $V_r/V_c$ is called Modulation Index (MI) and it controls the harmonic content of the output voltage waveform. The magnitude of fundamental component of output voltage is proportional to MI, but MI can never be more than unity. Thus the output voltage is controlled by varying MI.

Harmonic analysis of the output modulated voltage wave reveals that SPWM has the following important features:

(i) For MI less than one i.e. MI<1; largest harmonics amplitudes in the output voltage are associated with harmonics of higher order $(fc/f+1),(fc/f-1)$ or $(2N-1)$, where N is the number of pulse per half cycle. Thus, by increasing the number of pulses per half cycle, the order of dominant harmonic frequency can be raised, which can be filtered out easily. In fig.8 N=5, therefore harmonics of order 9 and 11 become significant in the output voltage. It may be noted that the highest order of significant harmonic of a modulated voltage wave is centered around the carrier frequency $f_c$.

It is observed from above that as N is increased, the order of significant harmonic increases and the filtering requirements are accordingly minimized. But higher value of N entails higher switching frequency of the thyristors. This amount higher switching losses and therefore an impaired inverter efficiency. Thus a compromise between the filtering requirements and inverter efficiency should be made.

(ii) For MI greater than one i.e. MI>1; lower order harmonics appear, pulse width are no longer a sinusoidal function.

This PWM signal can then be used to control switches connected to a high-voltage bus, which will replicate this signal at the appropriate voltage. Put through an LC filter, this SPWM signal will clean up into a close approximation of a sine wave. Though this technique produces a much cleaner source of AC power than either the square or modified sine waves, the frequency analysis shows that the primary harmonic is still truncated, and there is a relatively high amount of higher level harmonics in the signal.
f) **Space Vector PWM:** The space vector PWM (SVM) method is an advanced, computation-intensive PWM method and is possibly the best method among the all PWM techniques for variable-frequency drive application. Because of its superior performance characteristics, it has been finding widespread application in recent years.

**Space vector modulation (SVM)** is an algorithm for the control of pulse width modulation (PWM). It is used for the creation of alternating current (AC) waveforms; most commonly to drive 3 phase AC powered motors at varying speeds from DC using multiple class-D amplifiers. There are various variations of SVM that result in different quality and computational requirements. One active area of development is in the reduction of total harmonic distortion (THD) created by the rapid switching inherent to these algorithms.

To implement space vector modulation a reference signal $V_{\text{ref}}$ is sampled with a frequency $f_s$ ($T_s = 1/f_s$). The reference signal may be generated from three separate phase references using the $\alpha/\beta/\gamma$ transform. The reference vector is then synthesized using a combination of the two adjacent active switching vectors and one or both of the zero vectors. Various strategies of selecting the order of the vectors and which zero vector(s) to use exist. Strategy selection will affect the harmonic content and the switching losses.

![Basic inverter circuit and its waveform](image)

**Fig. Basic inverter circuit and its waveform**
Principle of Space Vector PWM:

The circuit model of a typical three-phase voltage source PWM inverter is shown in Fig. S1 to S6 are the six power switches that shape the output, which are controlled by the switching variables a, b, c and c. When an upper transistor is switched on, i.e., when a, b or c is 1, the corresponding lower transistor is switched off, i.e., the corresponding a’, b’ or c is 0. Therefore, the on and off states of the upper transistors S1, S3 and S5 can be used to determine the output voltage.

Fig.- Three-phase voltage source PWM Inverter.

The relationship between the switching variable vector \([a, b, c]^T\) and the line-to-line voltage vector \([V_{ab} V_{bc} V_{ca}]^T\) is given by in the following:

\[
\begin{bmatrix}
V_{ab} \\
V_{bc} \\
V_{ca}
\end{bmatrix} = V_{dc} \begin{bmatrix}
1 & -1 & 0 \\
0 & 1 & -1 \\
-1 & 0 & 1
\end{bmatrix} \begin{bmatrix}
a \\
b \\
c
\end{bmatrix} \quad \text{........................(1)}
\]
Also, the relationship between the switching variable vector \([a, b, c]t\) and the phase voltage vector \([V_a V_b V_c]t\) can be expressed below.

\[
\begin{bmatrix}
V_{an} \\
V_{bn} \\
V_{cn}
\end{bmatrix} = \frac{V_{dc}}{3} \begin{bmatrix}
2 & -1 & -1 \\
-1 & 2 & -1 \\
-1 & -1 & 2
\end{bmatrix} \begin{bmatrix}
a \\
b \\
c
\end{bmatrix}
\]

As illustrated in Fig. 1, there are eight possible combinations of on and off patterns for the three upper power switches. The on and off states of the lower power devices are opposite to the upper one and so are easily determined once the states of the upper power transistors are determined. According to equations (1) and (2), the eight switching vectors, output line to neutral voltage (phase voltage), and output line-to-line voltages in terms of DC-link Vdc, are given in Table 1 and Fig. 2 shows the eight inverter voltage vectors (V0 to V7).

<table>
<thead>
<tr>
<th>Voltage Vectors</th>
<th>Switching Vectors</th>
<th>Line to neutral voltage</th>
<th>Line to line voltage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(a) (b) (c)</td>
<td>(V_{an}) (V_{bn}) (V_{cn})</td>
<td>(V_{ab}) (V_{bc}) (V_{ca})</td>
</tr>
<tr>
<td>(V_0)</td>
<td>0 0 0</td>
<td>0 0 0</td>
<td>0 0 0</td>
</tr>
<tr>
<td>(V_1)</td>
<td>1 0 0</td>
<td>2/3 -1/3 -1/3</td>
<td>1 0 -1</td>
</tr>
<tr>
<td>(V_2)</td>
<td>1 1 0</td>
<td>1/3 1/3 -2/3</td>
<td>0 1 -1</td>
</tr>
<tr>
<td>(V_3)</td>
<td>0 1 0</td>
<td>-1/3 2/3 -1/3</td>
<td>-1 1 0</td>
</tr>
<tr>
<td>(V_4)</td>
<td>0 1 1</td>
<td>-2/3 1/3 1/3</td>
<td>-1 0 1</td>
</tr>
<tr>
<td>(V_5)</td>
<td>0 0 1</td>
<td>-1/3 -1/3 2/3</td>
<td>0 -1 1</td>
</tr>
<tr>
<td>(V_6)</td>
<td>1 0 1</td>
<td>1/3 -2/3 1/3</td>
<td>1 -1 0</td>
</tr>
<tr>
<td>(V_7)</td>
<td>1 1 1</td>
<td>0 0 0</td>
<td>0 0 0</td>
</tr>
</tbody>
</table>

(Note that the respective voltage should be multiplied by \(V_{dc}\).)

Table- Switching vectors, phase voltages and output line to line voltages
Space Vector PWM (SVPWM) refers to a special switching sequence of the upper three power transistors of a three-phase power inverter. It has been shown to generate less harmonic distortion in the output voltages and or currents applied to the phases of an AC motor and to provide more efficient use of supply voltage compared with sinusoidal modulation technique as shown in Fig.

Fig. - The eight inverter voltage vectors (V0 to V7)
To implement the space vector PWM, the voltage equations in the $abc$ reference frame can be transformed into the stationary $dq$ reference frame that consists of the horizontal (d) and vertical (q) axes as depicted in Fig.

Fig. - Locus comparison of maximum linear control voltage in Sine PWM and SVPWM

Fig.- The relationship of abc reference frame and stationary dq reference frame.
From this figure, the relation between these two reference frames is below

\[ \mathbf{f}_{dq0} = \mathbf{K}_s \mathbf{f}_{abc} \] 

where,

\[ \mathbf{K}_s = \frac{2}{3} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \]

\[ \mathbf{f}_{dq0} = [f_a, f_q, f_0]^T \]

\[ \mathbf{f}_{abc} = [f_a, f_b, f_c]^T \]

and \( f \) denotes either a voltage or a current variable.

As described in Fig. 4, this transformation is equivalent to an orthogonal projection of \( [a, b, c]^T \) onto the two-dimensional perpendicular to the vector \([1,1,1]^T\) (the equivalent d-q plane) in a three-dimensional coordinate system. As a result, six non-zero vectors and two zero vectors are possible. Six nonzero vectors (V1 - V6) shape the axes of a hexagonal as depicted in Fig. and feed electric power to the load. The angle between any adjacent two non-zero vectors is 60 degrees. Meanwhile, two zero vectors (V0 and V7) are at the origin and apply zero voltage to the load. The eight vectors are called the basic space vectors and are denoted by V0, V1, V2, V3, V4, V5, V6, and V7. The same transformation can be applied to the desired output voltage to get the desired reference voltage vector \( V_{ref} \) in the d-q plane.

The objective of space vector PWM technique is to approximate the reference voltage vector \( V_{ref} \) using the eight switching patterns. One simple method of approximation is to generate the average output of the inverter in a small period, \( T \) to be the same as that of \( V_{ref} \) in the same period.
Therefore, space vector PWM can be implemented by the following steps:

- **Step 1.** Determine $V_d$, $V_q$, $V_{ref}$, and angle ($\alpha$)
- **Step 2.** Determine time duration $T_1$, $T_2$, $T_0$
- **Step 3.** Determine the switching time of each transistor (S1 to S6)

**Step 1: Determine $V_d$, $V_q$, $V_{ref}$, and angle ($\alpha$)**

From Fig. the $V_d$, $V_q$, $V_{ref}$, and angle ($\alpha$) can be determined as follows:

$$V_d = V_{an} - V_{bn}.\cos60 - V_{cn}.\cos60$$

$$= V_{an} - \frac{1}{2}V_{bn} - \frac{1}{2}V_{cn}$$
\[ V_q = 0 + V_{bn}.\cos30 - V_{cn}.\cos30 \]
\[ = V_{an} + \frac{\sqrt{3}}{2} V_{bn} - \frac{\sqrt{3}}{2} V_{cn} \]

\[ \therefore \begin{bmatrix} V_d \\ V_q \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} \]

\[ \therefore |\bar{V}_{ref}| = \sqrt{V_d^2 + V_q^2} \]

\[ \therefore \alpha = \tan^{-1}\left(\frac{V_q}{V_d}\right) = \cot f = 2\pi f t, \]

Where, \( f \) = fundamental frequency.

Fig. Voltage Space Vector and its components in (d, q).
Step 2: Determine time duration $T_1$, $T_2$, $T_0$

From Fig. the switching time duration can be calculated as follows:

- **Switching time duration at Sector 1**

\[
\begin{align*}
    \int_0^{T_2} \overrightarrow{V}_{\text{ref}} \, dt &= \int_0^{T_1} \overrightarrow{V}_1 \, dt + \int_{T_1}^{T_1+T_2} \overrightarrow{V}_2 \, dt + \int_{T_1+T_2}^{T_2} \overrightarrow{V}_0 \, dt \\
    \therefore T_2 \cdot \overrightarrow{V}_{\text{ref}} &= (T_1 \cdot \overrightarrow{V}_1 + T_2 \cdot \overrightarrow{V}_2) \\
    \Rightarrow T_2 \cdot |\overrightarrow{V}_{\text{ref}}| \cdot \begin{bmatrix} \cos(\alpha) \\ \sin(\alpha) \end{bmatrix} &= T_1 \cdot \frac{2}{3} \cdot V_{dc} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} + T_2 \cdot \frac{2}{3} \cdot V_{dc} \cdot \begin{bmatrix} \cos(\pi/3) \\ \sin(\pi/3) \end{bmatrix} \\
    \text{(where, } 0 \leq \alpha \leq 60^\circ) \end{align*}
\]

\[
\therefore T_1 = T_2 \cdot \frac{\sin(\pi/3 - \alpha)}{\sin(\pi/3)}
\]

\[
\therefore T_2 = T_2 \cdot \frac{\sin(\alpha)}{\sin(\pi/3)}
\]

\[
\therefore T_0 = T_2 - (T_1 + T_2), \quad \begin{cases} \text{where, } & T_z = \frac{1}{f_z} \quad \text{and} \quad \alpha = \frac{\overrightarrow{V}_{\text{ref}}}{2 \cdot V_{dc}} \end{cases}
\]

Fig. Reference vector as a combination of adjacent vectors at sector 1.
Switching time duration at any Sector:

\[ T_1 = \frac{\sqrt{3} \cdot T_z \cdot |V_{\text{ref}}|}{V_{dc}} \left( \sin \left( \frac{\pi}{3} - \frac{n-1}{3} \pi \right) \right) \]

\[ = \frac{\sqrt{3} \cdot T_z \cdot |V_{\text{ref}}|}{V_{dc}} \left( \sin \frac{n}{3} \pi - \alpha \right) \]

\[ = \frac{\sqrt{3} \cdot T_z \cdot |V_{\text{ref}}|}{V_{dc}} \left( \sin \frac{n}{3} \pi \cos \alpha - \cos \frac{n}{3} \pi \sin \alpha \right) \]

\[ T_2 = \frac{\sqrt{3} \cdot T_z \cdot |V_{\text{ref}}|}{V_{dc}} \left( \sin \left( \alpha - \frac{n-1}{3} \pi \right) \right) \]

\[ T_3 = \frac{\sqrt{3} \cdot T_z \cdot |V_{\text{ref}}|}{V_{dc}} \left( \sin \left( \alpha - \frac{n-1}{3} \pi \right) \right) \]

\[ = \frac{\sqrt{3} \cdot T_z \cdot |V_{\text{ref}}|}{V_{dc}} \left( -\cos \alpha \cdot \sin \frac{n-1}{3} \pi + \sin \alpha \cdot \cos \frac{n-1}{3} \pi \right) \]

\[ T_0 = T_z - T_1 - T_2, \quad \text{where, } n = 1 \text{ through } 6 \text{ (that is, Sector1 to 6)} \]

\[ 0 \leq \alpha \leq 60^\circ \]
Step 3: Determine the switching time of each transistor (S1 to S6):

Sector 1

Sector 2

Sector 3

Sector 4

Sector 5

Sector 6

Fig. Space Vector PWM switching patterns at each sector
Based on Fig. 8, the switching time at each sector is summarized in Table 2, and it will be built in Simulink model to implement SVPWM.

<table>
<thead>
<tr>
<th>Sector</th>
<th>Upper Switches (S₁, S₃, S₅)</th>
<th>Lower Switches (S₄, S₆, S₂)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>S₁ = T₁ + T₂ + T₀ / 2</td>
<td>S₄ = T₀ / 2</td>
</tr>
<tr>
<td></td>
<td>S₃ = T₂ + T₀ / 2</td>
<td>S₆ = T₁ + T₀ / 2</td>
</tr>
<tr>
<td></td>
<td>S₆ = T₀ / 2</td>
<td>S₂ = T₁ + T₂ + T₀ / 2</td>
</tr>
<tr>
<td>2</td>
<td>S₁ = T₁ + T₀ / 2</td>
<td>S₄ = T₂ + T₀ / 2</td>
</tr>
<tr>
<td></td>
<td>S₃ = T₁ + T₂ + T₀ / 2</td>
<td>S₆ = T₀ / 2</td>
</tr>
<tr>
<td></td>
<td>S₅ = T₀ / 2</td>
<td>S₂ = T₁ + T₂ + T₀ / 2</td>
</tr>
<tr>
<td>3</td>
<td>S₁ = T₀ / 2</td>
<td>S₄ = T₁ + T₂ + T₀ / 2</td>
</tr>
<tr>
<td></td>
<td>S₃ = T₁ + T₂ + T₀ / 2</td>
<td>S₆ = T₀ / 2</td>
</tr>
<tr>
<td></td>
<td>S₅ = T₀ / 2</td>
<td>S₂ = T₁ + T₀ / 2</td>
</tr>
<tr>
<td>4</td>
<td>S₁ = T₀ / 2</td>
<td>S₄ = T₁ + T₂ + T₀ / 2</td>
</tr>
<tr>
<td></td>
<td>S₃ = T₁ + T₀ / 2</td>
<td>S₆ = T₂ + T₀ / 2</td>
</tr>
<tr>
<td></td>
<td>S₅ = T₁ + T₂ + T₀ / 2</td>
<td>S₂ = T₀ / 2</td>
</tr>
<tr>
<td>5</td>
<td>S₁ = T₂ + T₀ / 2</td>
<td>S₄ = T₁ + T₀ / 2</td>
</tr>
<tr>
<td></td>
<td>S₃ = T₀ / 2</td>
<td>S₆ = T₁ + T₂ + T₀ / 2</td>
</tr>
<tr>
<td></td>
<td>S₅ = T₁ + T₂ + T₀ / 2</td>
<td>S₂ = T₀ / 2</td>
</tr>
<tr>
<td>6</td>
<td>S₁ = T₁ + T₂ + T₀ / 2</td>
<td>S₄ = T₀ / 2</td>
</tr>
<tr>
<td></td>
<td>S₃ = T₀ / 2</td>
<td>S₆ = T₁ + T₂ + T₀ / 2</td>
</tr>
<tr>
<td></td>
<td>S₅ = T₁ + T₀ / 2</td>
<td>S₂ = T₂ + T₀ / 2</td>
</tr>
</tbody>
</table>

Table- Switching time calculation at each sector
State-Space Model:

Fig. shows L-C output filter to obtain current and voltage equations.

By applying Kirchoff’s current law to nodes a, b, and c, respectively, the following current equations are derived:

node “a”:

\[ i_{La} + i_{ca} = i_{ab} + i_{LA} \quad \Rightarrow \quad i_{La} + C_f \frac{dV_{LCA}}{dt} = C_f \frac{dV_{LAB}}{dt} + i_{LA} \quad \text{.........}(4) \]

node “b”:

\[ i_{IB} + i_{ab} = i_{bc} + i_{LB} \quad \Rightarrow \quad i_{IB} + C_f \frac{dV_{LAB}}{dt} = C_f \frac{dV_{LBC}}{dt} + i_{LB} \quad \text{.........}(5) \]

node “c”:

\[ i_{IC} + i_{bc} = i_{ca} + i_{LC} \quad \Rightarrow \quad i_{IC} + C_f \frac{dV_{LBC}}{dt} = C_f \frac{dV_{LCA}}{dt} + i_{LC} \quad \text{.........}(6) \]
Where,

\[ i_{ab} = C_f \frac{dV_{LAB}}{dt}, \quad i_{bc} = C_f \frac{dV_{LBC}}{dt}, \quad i_{ca} = C_f \frac{dV_{LCA}}{dt}. \]

Also, (4) to (6) can be rewritten as the following equations, respectively:

**subtracting (5) from (4):**

\[ i_A - i_B + C_f \left( \frac{dV_{LCA}}{dt} - \frac{dV_{LAB}}{dt} \right) = C_f \left( \frac{dV_{LAB}}{dt} - \frac{dV_{LBC}}{dt} \right) + i_A - i_B \]

\[ \Rightarrow C_f \left( \frac{dV_{LCA}}{dt} + \frac{dV_{LBC}}{dt} - 2 \cdot \frac{dV_{LAB}}{dt} \right) = -i_A + i_B + i_A - i_B \]

\[ \text{.......(7)} \]

**subtracting (6) from (5):**

\[ i_B - i_C + C_f \left( \frac{dV_{LAB}}{dt} - \frac{dV_{LBC}}{dt} \right) = C_f \left( \frac{dV_{LBC}}{dt} - \frac{dV_{LCA}}{dt} \right) + i_B - i_C \]

\[ \Rightarrow C_f \left( \frac{dV_{LAB}}{dt} + \frac{dV_{LCA}}{dt} - 2 \cdot \frac{dV_{LBC}}{dt} \right) = -i_B + i_C + i_B - i_C \]

\[ \text{.......(8)} \]

**subtracting (4) from (6):**

\[ i_C - i_A + C_f \left( \frac{dV_{LBC}}{dt} - \frac{dV_{LCA}}{dt} \right) = C_f \left( \frac{dV_{LCA}}{dt} - \frac{dV_{LAB}}{dt} \right) + i_C - i_A \]

\[ \Rightarrow C_f \left( \frac{dV_{LAB}}{dt} + \frac{dV_{LBC}}{dt} - 2 \cdot \frac{dV_{LCA}}{dt} \right) = -i_C + i_A + i_C - i_A \]

\[ \text{.......(9)} \]
To simplify (7) to (9), we use the following relationship that an algebraic sum of line to line load voltages is equal to zero:

\[ V_{LAB} + V_{LBC} + V_{LCA} = 0 \] ............(10)

Based on (10), the (7) to (9) can be modified to a first-order differential equation, respectively:

\[
\begin{align*}
\frac{dV_{LAB}}{dt} &= \frac{1}{3C_f} i_{LAB} - \frac{1}{3C_f} (i_{LAB}) \\
\frac{dV_{LBC}}{dt} &= \frac{1}{3C_f} i_{LBC} - \frac{1}{3C_f} (i_{LBC}), \\
\frac{dV_{LCA}}{dt} &= \frac{1}{3C_f} i_{LCA} - \frac{1}{3C_f} (i_{LCA})
\end{align*}
\] ............(11)

Where,

\[ i_{LAB} = i_A - i_B, \quad i_{LBC} = i_B - i_C, \quad i_{LCA} = i_C - i_A \] and \[ i_{LAB} = i_A - i_B, \quad i_{LBC} = i_B - i_C, \quad i_{LCA} = i_C - i_A. \]

By applying Kirchoff’s voltage law on the side of inverter output, the following voltage equations can be derived:

\[
\begin{align*}
\frac{di_{LAB}}{dt} &= -\frac{1}{L_f} V_{LAB} + \frac{1}{L_f} V_{LAB} \\
\frac{di_{LBC}}{dt} &= -\frac{1}{L_f} V_{LBC} + \frac{1}{L_f} V_{LBC} \\
\frac{di_{LCA}}{dt} &= -\frac{1}{L_f} V_{LCA} + \frac{1}{L_f} V_{LCA}
\end{align*}
\] ............(12)
By applying Kirchoff’s voltage law on the load side, the following voltage equations can be derived:

\[
\begin{align*}
V_{LAB} &= L_{load} \frac{di_{LAB}}{dt} + R_{load} i_{LAB} - L_{load} \frac{di_{LB}}{dt} - R_{load} i_{LB} \\
V_{LBC} &= L_{load} \frac{di_{LB}}{dt} + R_{load} i_{LB} - L_{load} \frac{di_{LC}}{dt} - R_{load} i_{LC} \\
V_{LCA} &= L_{load} \frac{di_{LC}}{dt} + R_{load} i_{LC} - L_{load} \frac{di_{LA}}{dt} - R_{load} i_{LA} \\
\end{align*}
\]

\[\cdots\cdots(13)\]

Equation (13) can be rewritten as:

\[
\begin{align*}
\frac{di_{LAB}}{dt} &= -\frac{R_{load}}{L_{load}} i_{LAB} + \frac{1}{L_{load}} V_{LAB} \\
\frac{di_{LBC}}{dt} &= -\frac{R_{load}}{L_{load}} i_{LBC} + \frac{1}{L_{load}} V_{LBC} \\
\frac{di_{LCA}}{dt} &= -\frac{R_{load}}{L_{load}} i_{LCA} + \frac{1}{L_{load}} V_{LCA} \\
\end{align*}
\]

\[\cdots\cdots(14)\]

Therefore, we can rewrite (11), (12) and (14) into a matrix form, respectively:

\[
\begin{align*}
\frac{dV_L}{dt} &= \frac{1}{3C_f} I_i - \frac{1}{3C_f} I_L \\
\frac{dI_i}{dt} &= -\frac{1}{L_f} V_L + \frac{1}{L_f} V_i \\
\frac{dI_L}{dt} &= \frac{1}{L_{load}} V_L - \frac{R_{load}}{L_{load}} I_L \\
\end{align*}
\]

\[\cdots\cdots(15)\]

where, \( V_L = [V_{LAB} V_{LBC} V_{LCA}]^T \), \( I_i = [i_{LAB} i_{LBC} i_{LCA}]^T = [i_{iA} - i_{iB} i_{iB} - i_{iC} i_{iC} - i_{iA}]^T \), \( V_i = [V_{LAB} V_{LBC} V_{LCA}]^T \), \( I_L = [i_{LAB} i_{LBC} i_{LCA}]^T = [i_{iA} - i_{iB} i_{iB} - i_{iC} i_{iC} - i_{iA}]^T \).
Finally, the given plant model (15) can be expressed as the following continuous-time state space equation:

\[ \dot{X}(t) = AX(t) + Bu(t), \]  

Where,

\[ X = \begin{bmatrix} V_L \\ I_f \\ I_L \end{bmatrix}_{9 \times 1}, \]

\[ u = \begin{bmatrix} V_i \end{bmatrix}_{3 \times 1}, \]

\[ A = \begin{bmatrix} 0_{3 \times 3} & \frac{1}{3C_f}I_{3 \times 3} & \frac{1}{3C_f}I_{3 \times 3} \\ -\frac{1}{L_f}I_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \\ -\frac{1}{L_{load}}I_{3 \times 3} & 0_{3 \times 3} & -\frac{R_{load}}{L_{load}}I_{3 \times 3} \end{bmatrix}_{9 \times 9}, \]

\[ B = \begin{bmatrix} 0_{3 \times 3} \\ \frac{1}{L_f}I_{3 \times 3} \\ 0_{3 \times 3} \end{bmatrix}_{9 \times 3}. \]

Note that load line to line voltage \( V_L \), inverter output current \( I_i \), and the load current \( I_L \) are the state variables of the system, and the inverter output line-to-line voltage \( V_i \) is the control input \( u \).
Linear or Under Modulation Region \((0 < m < 0.907)\)

Let us first consider the linear or undermodulation region where the inverter transfer characteristics are naturally linear. The modulating command voltages of a three-phase inverter are always sinusoidal, and therefore, they constitute a rotating space vector \(\overline{V}^*\), as shown in figure (a) & figure (b) shows the phase a component of the reference wave on the six step phase voltage wave profile. For the location of the \(\overline{V}^*\) vector shown in figure (a), as an example, a convenient way to generate the PWM output is to use the adjacent vectors \(\overline{V}_1\) and \(\overline{V}_2\) of sector 1 on a part-time basis to satisfy the average output demand. The \(\overline{V}^*\) can be resolved as (dropping bar)

\[
V_a^* \sin \left(\frac{\pi}{3} - \alpha\right) = V_a \sin \frac{\pi}{3}
\]

\[
V_b^* \sin \alpha = V_b \sin \frac{\pi}{3}
\]

that is,

\[
V_a = \frac{2}{\sqrt{3}} V_a^* \sin \left(\frac{\pi}{3} - \alpha\right)
\]

\[
V_b = \frac{2}{\sqrt{3}} V_b^* \sin \alpha
\]

![Fig- (a) space vectors (bar sign is omitted) of three-phase bridge inverter showing reference voltage trajectory and segments of adjacent voltage vectors](attachment:image.png)
Where $V_a$ and $V_b$ are the components of $V^*$ aligned in the directions of $V_1$ and $V_2$, respectively. Considering the period $T_c$ during which the average output should match the command, we can write the vector addition

$$V^* = V_a + V_b = V_1 \frac{t_a}{T_c} + V_2 \frac{t_b}{T_c} + (V_o \text{ or } V_7) \frac{t_o}{T_c}$$

Or

$$V^* T_c = V_1 t_a + V_2 t_b + (V_o \text{ or } V_7) t_o$$

where

$$t_a = \frac{V_a}{V_1} T_c$$

$$t_b = \frac{V_b}{V_2} T_c$$

$$t_o = T_c - (t_a + t_b)$$

Note that the time intervals $t_a$ and $t_b$ satisfy the command voltage, but time $t_o$ fills up the remaining gap for $T_c$ with the zero or null vector. Here, $T_s = 2T_c = 1/fs$ ($fs$ = switching frequency) is the sampling time. Note the null time has been conveniently distributed between the $V_0$ and $V_7$ vectors to describe the symmetrical pulse widths. Studies have shown that a symmetrical pulse pattern gives minimal output harmonics.
Modulation index:

\[ m' = \frac{\hat{V}^*}{\hat{V}_{1,sw}} \]

Where \( V^* \) = vector magnitude or phase peak value & \( V_{1,sw} \) = fundamental peak value of the square-phase voltage wave.

The radius of the inscribed circle can be given as-

\[ V_{m}^* = \frac{2}{3} V_d \cos \frac{\pi}{6} = 0.577V_d \]

Therefore,

\[ m' = \frac{\hat{V}_{m}^*}{\hat{V}_{1,sw}} = \frac{0.577V_d}{2/\pi \cdot V_d} = 0.907 \]
Development of MATLAB Simulink model for SVPWM with under-modulation region:
Simulation:

Simulation Steps:

- Initialize system parameters using Matlab
- Build simulink model
  - Determine sector
  - Determine time duration $T_1$, $T_2$, $T_0$
  - Determine the switching time ($T_a$, $T_b$, and $T_c$) of each transistor ($S_1$ to $S_6$)
  - Generate the inverter output voltages ($V_{iAB}$, $V_{iBC}$, $V_{iCA}$) for control input ($u$)
  - Send data to Workspace
- Plot simulation results using Matlab

Simulation Results:

![Three phase waveform diagram](image_url)

Simulation block to generate the three phase
Simulation block diagram & angle and magnitude of rotating space vector
Generation of 3 phase after modulation

$V_{an}$

Fourier analysis of the phase voltage a
Fourier analysis of the phase voltage $b$

$V_{bn}$

Fourier analysis of the phase voltage $c$

$V_{cn}$
Fig. Simulation results of inverter output line to line voltages (\(V_{iAB}, V_{iBC}, V_{iCA}\))
Fig. Simulation results of inverter output currents (i_A, i_B, i_C)
Fig. Simulation results of load line to line voltages (VLAB, VLBC, VLCA)
Fig. Simulation results of load phase currents (iLA, iLB, iLC)
Fig. 14 Simulation waveforms.
(a) Inverter output line to line voltage (ViAB)
(b) Inverter output current (iIA)
(c) Load line to line voltage (VLAB)
(d) Load phase current (iLA)
IGBT based 50V 3-phase Inverter with PSIM
Graph: Output Voltage - Time

Vs = Carrier Voltage, Vt = Modulated Voltage

Graph: Vt and Vs – Time
Conclusion and Future Work:

- The simulation of modulation of SVPWM is carried out in MATLAB/Simulink. Modulation index is 0.905; the output voltage magnitude at wanted frequency (50 Hz) is nearly 230.7 V. It is in the under-modulation region (0< MI<0.906).

- If the modulation index is 0.942 and then corresponding magnitude is 240 V. This is called over modulation region (0.906<MI<1).

- In SVPWM 90.7% of fundamental at the square wave is available in the linear region, compare to 78.55% in the sinusoidal PWM.

- The current and torque harmonics produced are much less in case of SVPWM.

However despite all the above mentioned advantages that SVPWM enjoys over SPWM, SVPWM algorithm used in three-level inverters is more complex because of large number of inverter switching states.

The presented simulink model can be setup experimentally using semiconductor devices like Transistor, Thyristor, MOSFET etc., and controlling strategies by using any of the following devices like microprocessor, microcontroller, Digital Signal Processor and FPGA. The controlling strategies can be implemented using m-file programming with FPGA for better processing speed and performance.
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- PSIM (software)